# Pharmaceutical statistics

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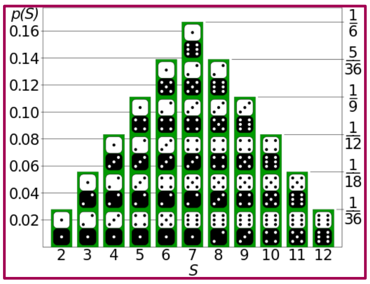
ARKAN ACADEMY

# Probability Distributions

- The probability distribution of random variables is a *graph*, *table*, or *formula* used to specify all possible values of a random variable along with the respective probabilities.




$$f(z_i) = \frac{1}{(2\pi)^{1/2}} e^{-z_i^2/2}$$

$Z_i$: Std. Score utk kes i variabel x

$e = 2.718$

$\pi = 3.141$

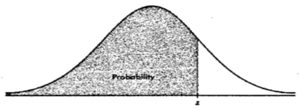



TABLE A: STANDARD NORMAL PROBABILITIES (CONTINUED)

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

- ***Probability Distributions example:***
  - The following table shows the *prevalence* of prescription and nonprescription drugs use in pregnancy among a population of women who were delivered of infants.
  - We wish to construct the probability distribution of the variable X = number of prescription and nonprescription drugs used by the study subjects.

Variable Probability

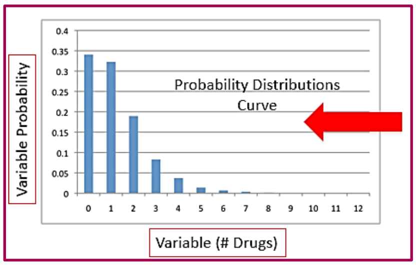
Variable

Probability Distributions Table

| # Drugs | Frequency | Relative Frequency (Probability) |
|---|---|---|
| 0 | 1425 | 0.340502 |
| 1 | 1351 | 0.32282 |
| 2 | 793 | 0.189486 |
| 3 | 348 | 0.083154 |
| 4 | 156 | 0.037276 |
| 5 | 58 | 0.013859 |
| 6 | 28 | 0.006691 |
| 7 | 15 | 0.003584 |
| 8 | 6 | 0.001434 |
| 9 | 3 | 0.000717 |
| 10 | 1 | 0.000239 |
| 11 | 0 | 0 |
| 12 | 1 | 0.000239 |
| **Total** | **4185** | **1** |

Note: $0 \le P(X=x) \le 1$; $\Sigma P(X=x)=1$

  - *Q1*: What is the probability that a randomly selected woman will be one who used exactly three prescription and nonprescription drugs?
    P(x=3) = 0.0832
  - *Q2*: What is the probability that a randomly selected woman used wither one or two drugs? (We use the addition rule for mutually exclusive events)
    P (1 ∪ 2) = P (1) + P (2) = 0.3228 + 0.1895 = 0.5123
  - *Q3*: What is the probability that a randomly selected woman used two and three drugs? Zero




- Sometimes we may want to work with a *cumulative probability* distribution of a random variable.
- The cumulative probability distribution is obtained by successively adding the probabilities, P(X=x) given in the last column of the previous table.

| # Drugs | Frequency | Cumulative Frequency | Relative Cumulative Frequency (**Cumulative Probability**) |
|---|---|---|---|
| 0 | 1425 | 1425 | 0.34050179 |
| 1 | 1351 | 2776 | 0.66332139 |
| 2 | 793 | 3569 | 0.85280765 |
| 3 | 348 | 3917 | 0.93596177 |
| 4 | 156 | 4073 | 0.97323775 |
| 5 | 58 | 4131 | 0.98709677 |
| 6 | 28 | 4159 | 0.99378734 |
| 7 | 15 | 4174 | 0.99737157 |
| 8 | 6 | 4180 | 0.99880526 |
| 9 | 3 | 4183 | 0.9995221 |
| 10 | 1 | 4184 | 0.99976105 |
| 11 | 0 | 4184 | 0.99976105 |
| 12 | 1 | 4185 | **1** |
| Total | **4185** | | |

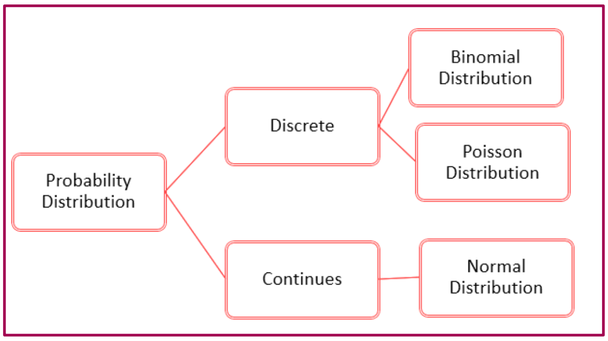
  - ✓ *Q3*: What is the probability that a woman picked at random will be one who used two or fewer drugs? P ($x \le 2$) =0.8528
  - ✓ *Q4*: What is the probability that a randomly selected woman will be one who used fewer than two drugs? P ($x<2$) = 0.6633

- *Q5*: What is the probability that a randomly selected woman is one who used between three and five drugs, inclusive?
  P ($x \le 5$) = 0.9872 is the probability that a woman used between zero and five drugs, inclusive.
  P ($x \le 2$) = 0.8528 is the probability that a woman used between zero and two drugs, inclusive.
  P ($3 \le x \le 5$) = P($x \le 5$) - P ($X \le 2$) = 0.9872 - 0.8528 =0.1344

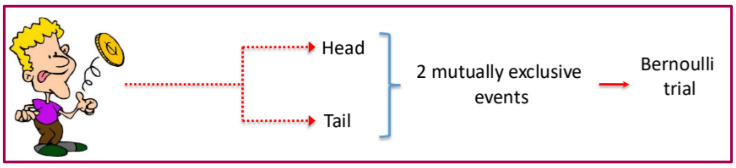
- *Q6*: What is the probability that a randomly selected woman is one who used at least 11 drugs?
  P ($x \ge 11$) = 1 - P ($x \le 10$) = 1 - 0.99976105 = 0.000239

- *Q7*: What is the probability that a randomly selected woman is one who used at 12 drugs?
  P (x=12) = 0.000239

- ***Probability Distributions (Types)***




- ***Binomial Distribution***
  - One of the most widely encountered discrete distributions in applied statistics.
  - It is derived from a process known as the Bernoulli trial (James Bernoulli, 1654-1705).
  - When a random process or experiment, called a trial, can result in two mutually exclusive outcomes, such as dead or alive, sick or well, male or female, the trial is called a Bernoulli trial.




- ***Repeated identical trials are called Bernoulli trials if:***
  - There are two possible mutually exclusive outcomes for each trial, denoted arbitrarily as success **(s)** and failure **(f).**
  - The trials are independent; that is, the outcome of any particular trial is *not affected* by the outcome of any other trial.
  - The probability of a success, denoted by ***p***, remains the same from trial to trial.
  - The probability of failure, ***1-p***, is denoted by ***q***.
- Example:
  - A drug is known to be 80% effective in curing a certain disease. Suppose that four patients are to be given the drug and the cure - no cure results recorded.
  - What is the probability of exactly 3 patients will be cured?
  - There are two possible outcomes for each trial: Success (cure) or failure (no cure).
  - The trials are independent (why?).
  - The success probability is ***p = 0.8*** (80%).
  - The failure probability is ***q =1- 0.8 = 0.2*** (20%)
  - All possible outcomes of the four Bernoulli trials (four patients taking the mentioned drug) are:

| | | | |
|---|---|---|---|
| *ssss* | *sfss* | *fsss* | *ffss* |
| *sssf* | *sfsf* | *fssf* | *ffsf* |
| *ssfs* | *sffs* | *fsfs* | *fffs* |
| *ssff* | *fsff* | *fsff* | *ffff* |

***All possible outcomes with their probability***

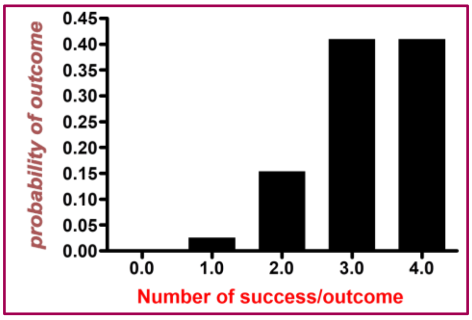
| | | | |
|---|---|---|---|
| *ssss* 0.4096 | *sfss* 0.1024 | *fsss* 0.1024 | *ffss* 0.0256 |
| *sssf* 0.1024 | *sfsf* 0.0256 | *fssf* 0.0256 | *ffsf* 0.0064 |
| *ssfs* 0.1024 | *sffs* 0.0256 | *fsfs* 0.0256 | *fffs* 0.0064 |
| *ssff* 0.0256 | *fsff* 0.0064 | *fsff* 0.0064 | *ffff* 0.0016 |

***Probability Distributions Table***

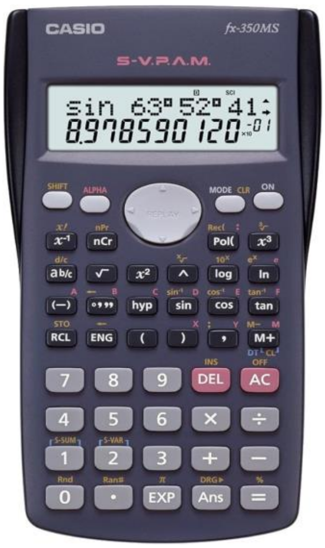
| Outcome | Probability |
|---|---|
| ssss | (0.8)(0.8)(0.8)(0.8) = 0.4096 |
| sssf | (0.8)(0.8)(0.8)(0.2) = 0.1024 |
| ssfs | (0.8)(0.8)(0.2)(0.8) = 0.1024 |
| ssff | (0.8)(0.8)(0.2)(0.2) = 0.0256 |
| sfss | (0.8)(0.2)(0.8)(0.8) = 0.1024 |
| sfsf | (0.8)(0.2)(0.8)(0.2) = 0.0256 |
| sffs | (0.8)(0.2)(0.2)(0.8) = 0.0256 |
| sfff | (0.8)(0.2)(0.2)(0.2) = 0.0064 |
| fsss | (0.2)(0.8)(0.8)(0.8) = 0.1024 |
| fssf | (0.2)(0.8)(0.8)(0.2) = 0.0256 |
| fsfs | (0.2)(0.8)(0.2)(0.8) = 0.0256 |
| fsff | (0.2)(0.8)(0.2)(0.2) = 0.0064 |
| ffss | (0.2)(0.2)(0.8)(0.8) = 0.0256 |
| ffsf | (0.2)(0.2)(0.8)(0.2) = 0.0064 |
| fffs | (0.2)(0.2)(0.2)(0.8) = 0.0064 |
| ffff | (0.2)(0.2)(0.2)(0.2) = 0.0016 |

- ✓ Sum of all outcome's probability is 1, why?
- ✓ The probability of each outcome is **not** necessary the same, why?
- ✓ p(sfss) = p(fsss) = p(sssf) = p(sfsf).
- ✓ P(sfss) = 0.8 * 0.2 * 0.8 * 0.8 = 0.1024
- ✓ Probability of **exactly 3** patients will be cured= P (sfss or fsss or sssf or sfsf) = P(sfss)+P(fsss)+P(sssf)+P(sfsf) = 0.1024+0.1024+0.1024+0.1024 = 4 * 0.1024= 0.4096

| Number of success/ outcome | Probability of outcome |
|---|---|
| 0 | 0.0016 |
| 1 | 0.0256 |
| 2 | 0.1536 |
| 3 | 0.4096 |
| 4 | 0.4096 |
| | 1.0 |




- As the size of the sample increases, listing the number of sequences becomes more and more difficult and tedious.
  - ➢ An easy method for counting the number of sequences is provided by means of a counting formula to obtain the *probability of observing x successes in n trials*
  - ➢ $\boldsymbol{P(x) = \binom{n}{x} p^x (1-p)^{n-x}}$
  - ➢ $\binom{n}{x}$ is the Binomial coefficient and is equal to $\frac{n!}{x!(n-x)!}$ **Or on calculator**




★ Let's solve the example above using the equation:

- ✓ x = 3, n = 4, p = 0.8, q = 0.2
- ✓ *P (3)* = $\binom{4}{3}$ $0.8^3$ $(1-0.8)^{4-3}$ = 4 * 0.512 * 0.2 = 0.4096
- ✓ Note: $\binom{n}{n-1} = n$ , $\binom{n}{1} = n$ , *0! = 1*

★ Example:

- ✓ x = 0, n = 4, p = 0.8, q = 0.2
- ✓ *P (0)* = $\binom{4}{0}$ $0.8^0$ $(1-0.8)^{4-0}$ = 0.0016

- ***Assumptions for using Binomial Distribution Formula:***
  - n identical trials are to be performed.
  - There are two possible outcomes (success and failure) for each trial.
  - The trials are independent.
  - The success probability, p, remains the same from trial to trial.

- ***Using Binomial Distribution Formula***
  1. ***Identify a success:*** **(S)**
  2. ***Determine the success probability:*** **(p)**
  3. ***Determine number of trials:*** **(n)**
  4. ***Determine number of successes in n trials:*** **(x)**

★ Example:

- ✓ Suppose that 30% of a certain population are immune to a certain disease. If a random sample of size 10 is to be selected from that population.
- ✓ What is the probability that it will contain exactly 4 immune persons?
  (S) Immune, (p) 0.3, (n) 10, (x) 4.
- ✓ $P\ (4) = \binom{10}{4}\ 0.3^{10}\ (1-0.3)^{10-4} = 0.2001$

- ***Probability Distributions***
  - The calculation of a probability using the previous equation can be a tedious undertaking if the sample size is large.
  - If he asked for cumulative probability, it becomes hard.
  - X≤ 4, n=10, p= 0.3, q= 0.7      p (4) + p (3) + p (2) + p (1)
  - Fortunately, probabilities for different values of n, p, and x have been **tabulated**, so that we need only to consult an appropriate table to obtain the desired probability.
  - The table gives the probability that x is *less than or equal to some specified value*. That is, the table gives the *cumulative probabilities from x = 0 up through some specified value*.
  - This table shows the probability of *x* successes in *n* independent Bernoulli trials, each with probability of success *P*

| | | | | | | | p | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| n | x | .01 | .05 | .10 | .15 | .20 | .25 | .30 | .35 | .40 | .45 | .50 |
| 2 | 0 | 0.9801 | 0.9025 | 0.8100 | 0.7225 | 0.6400 | 0.5625 | 0.4900 | 0.4225 | 0.3600 | 0.3025 | 0.2500 |
| | 1 | 0.9999 | 0.9975 | 0.9900 | 0.9775 | 0.9600 | 0.9375 | 0.9100 | 0.8775 | 0.8400 | 0.7975 | 0.7500 |
| | 2 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 3 | 0 | 0.97030 | 0.85738 | 0.729 | 0.61413 | 0.512 | 0.42187 | 0.343 | 0.27463 | 0.216 | 0.16638 | 0.125 |
| | 1 | 0.99970 | 0.99275 | 0.972 | 0.93925 | 0.896 | 0.84375 | 0.784 | 0.71825 | 0.648 | 0.57475 | 0.500 |
| | 2 | 1.00000 | 0.99988 | 0.999 | 0.99663 | 0.992 | 0.98437 | 0.973 | 0.95713 | 0.936 | 0.90887 | 0.875 |
| | 3 | 1.00000 | 1.00000 | 1.000 | 1.00000 | 1.000 | 1.00000 | 1.000 | 1.00000 | 1.000 | 1.00000 | 1.000 |

★ Example 1:

- ✓ Suppose that **30%** of a certain population are **immune** to a certain disease. If a random **sample of size 10** is to be selected from that population.
- ✓ ***Q: What is the probability that it will contain exactly 4 immune persons?***
  In other words: What is the probability that x = 4 when n = 10 and p = 0.3
  Drawing on our knowledge of cumulative probability distributions, we know that P(x = 4) may be found by subtracting $P(X \leq 3)$ from $P(X \leq 4)$.
- ✓ In the binomial distribution table below, we locate P = 0.3 for n = 10, we find that $P(X \leq 4)$ = 0.8497 and $P(X \leq 3)$ = 0.6496. Subtracting the latter from the former gives 0.8497 - 0.6496= 0.2001, which agrees with our hand calculation (previous lecture).

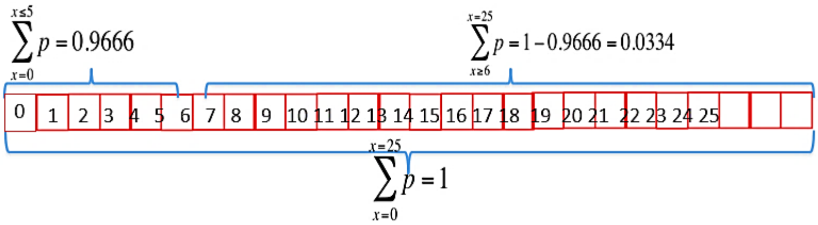
| | | p | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| n | x | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
| 10 | 0 | 0.599 | 0.349 | 0.197 | 0.107 | 0.056 | 0.028 | 0.013 | 0.006 | 0.003 | 0.001 |
| | 1 | 0.914 | 0.736 | 0.544 | 0.376 | 0.244 | 0.149 | 0.086 | 0.046 | 0.023 | 0.011 |
| | 2 | 0.988 | 0.930 | 0.820 | 0.678 | 0.526 | 0.383 | 0.262 | 0.167 | 0.100 | 0.055 |
| | 3 | 0.999 | 0.987 | 0.950 | 0.879 | 0.776 | 0.650 | 0.514 | 0.382 | 0.266 | 0.172 |
| | 4 | 1.000 | 0.998 | 0.990 | 0.967 | 0.922 | 0.850 | 0.751 | 0.633 | 0.504 | 0.377 |
| | 5 | 1.000 | 1.000 | 0.999 | 0.994 | 0.980 | 0.953 | 0.905 | 0.834 | 0.738 | 0.623 |
| | 6 | 1.000 | 1.000 | 1.000 | 0.999 | 0.996 | 0.989 | 0.974 | 0.945 | 0.898 | 0.828 |
| | 7 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.998 | 0.995 | 0.988 | 0.973 | 0.945 |

★ Example 2:

- ✓ Suppose it is known that in a certain population 10 percent of the population is color blind. If a random sample of 25 people is drawn from this population, use the Binomial table to find the probability that
- ✓ ***Q1: Five or fewer are color blind.***
  1. Identify a success: (S) color blind
  2. Determine the success probability: (p) 0.1
  3. Determine number of trials: (n) 25
  4. Determine number of successes in n trials: $(x \leq 5)$ This probability is an entry in the table. No addition or subtraction is necessary. $P(x \leq 5)$ = 0.9666

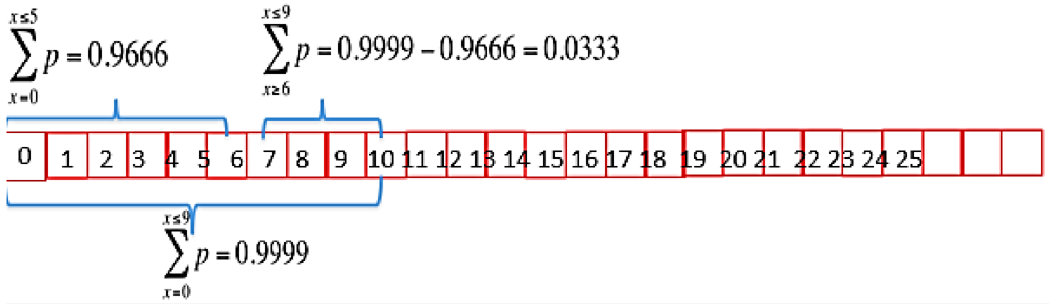
| | | p | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| n | x | .01 | .05 | .10 | .15 | .20 | .25 | .30 | .35 | .40 | .45 | .50 |
| 25 | 0 | 0.77782 | 0.27739 | 0.07179 | 0.01720 | 0.00378 | 0.00075 | 0.00013 | 0.00002 | 0.00000 | 0.00000 | 0.00000 |
| | 1 | 0.97424 | 0.64238 | 0.27121 | 0.09307 | 0.02739 | 0.00702 | 0.00157 | 0.00030 | 0.00005 | 0.00001 | 0.00000 |
| | 2 | 0.99805 | 0.87289 | 0.53709 | 0.25374 | 0.09823 | 0.03211 | 0.00896 | 0.00213 | 0.00043 | 0.00007 | 0.00001 |
| | 3 | 0.99989 | 0.96591 | 0.76359 | 0.47112 | 0.23399 | 0.09621 | 0.03324 | 0.00968 | 0.00237 | 0.00048 | 0.00008 |
| | 4 | 1.00000 | 0.99284 | 0.90201 | 0.68211 | 0.42067 | 0.21374 | 0.09047 | 0.03205 | 0.00947 | 0.00231 | 0.00046 |
| | 5 | 1.00000 | 0.99879 | 0.96660 | 0.83848 | 0.61669 | 0.37828 | 0.19349 | 0.08262 | 0.02936 | 0.00860 | 0.00204 |
| | 6 | 1.00000 | 0.99983 | 0.99052 | 0.93047 | 0.78004 | 0.56110 | 0.34065 | 0.17340 | 0.07357 | 0.02575 | 0.00732 |

- ✓ ***Q2: Six or more will be color blind***
  1. *Identify a success:* (S) color blind
  2. *Determine* the *success probability:* (p) 0.1
  3. *Determine* number *of* trials: (n) 25
  4. Determine number of successes in n trials: $(x \geq 6)$
- ✓ We *cannot* find this probability directly in the table.
- ✓ To find the answer, we use the *concept of complementary probabilities*. The probability that six or more are color blind is the complement of the probability that five or fewer are not color blind.
- ✓ $P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.9666 = 0.0334$




- ✓ ***Q3:*** ***Between six and nine inclusive will be color blind***
  1. *Identify a success:* (S) color blind
  2. *Determine* the *success probability:* (p) 0.1
  3. *Determine* number *of* trials: (n) 25
  4. Determine number of successes in n trials: **(6≤ x ≤9)**
- ✓ We find this by subtracting the probability that X is less than or equal to 5 from the probability that X is less than or equal to 9.

  P(6≤X≤9) = P(X≤9) - P(X≤5) = 0.9999 - 0.9666 = 0.0333




- ***What about if p >0.5??***
  - ➢ The table usually comes up to 0.5 p
  - ➢ We will calculate the probability of failure as if the probability of success ***p >0.5***, the probability of failure ***q < 0.5***
  - ➢ Binomial tables **<u>do not</u>** give probabilities for values of p greater than 0.5.
  - ➢ We may obtain probabilities from Binomial tables by **restating** the problem in terms of the probability of a **failure**, ***q=1 - p***, rather than in terms of the probability of a success, *p*.
  - ➢ As part of the restatement, we must also think in terms of the number of failures, **x`**=*n - x*, rather than the number of successes, x.
  - ➢ For purposes of using the binomial table we treat the probability of a failure as though it were the probability of a success. When p is greater than 0.5, we may obtain cumulative probabilities from Binomial tables by using the following relationships:

    1. P (X=x, n, p) = P (X=x`, n, q)
    2. P (X≤ x, n, p) = P (X≥ x`, n, q)
    3. P (X≥ x, n, p) = P (X≤ x`, n, q)

    **x`**= n - x

    **q**=1 – p

| n | x | .01 | .05 | .10 | .15 | .20 | .25 | .30 | .35 | .40 | .45 | .50 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 25 | 0 | 0.77782 | 0.27739 | 0.07179 | 0.01720 | 0.00378 | 0.00075 | 0.00013 | 0.00002 | 0.00000 | 0.00000 | 0.00000 |
| | 1 | 0.97424 | 0.64238 | 0.27121 | 0.09307 | 0.02739 | 0.00702 | 0.00157 | 0.00030 | 0.00005 | 0.00001 | 0.00000 |
| | 2 | 0.99805 | 0.87289 | 0.53709 | 0.25374 | 0.09823 | 0.03211 | 0.00896 | 0.00213 | 0.00043 | 0.00007 | 0.00001 |
| | 3 | 0.99989 | 0.96591 | 0.76359 | 0.47112 | 0.23399 | 0.09621 | 0.03324 | 0.00968 | 0.00237 | 0.00048 | 0.00008 |
| | 4 | 1.00000 | 0.99284 | 0.90201 | 0.68211 | 0.42067 | 0.21374 | 0.09047 | 0.03205 | 0.00947 | 0.00231 | 0.00046 |
| | 5 | 1.00000 | 0.99879 | 0.96660 | 0.83848 | 0.61669 | 0.37828 | 0.19349 | 0.08262 | 0.02936 | 0.00860 | 0.00204 |

- ➢ Our class has 70 students, and we selected 5 students randomly. Among the selected students:

1.The Probability to have 2 smokers=Probability to have 3 non-smokers
2. The probability to have less than 2 smokers=Probability to have more than 3 non-smokers
3. The probability to have more than 2 smokers=Probability to have less than 3 non-smokers

★ Example 1:

✓ In a certain community, on a given evening, someone is at home in *85 percent* of the households. A health research team conducting a telephone survey selects a random *sample of 12* households. Use binomial tables to find the probability that:

**1.** The team will find someone at home in exactly 7 households.
We restate the problem as follows: What is the probability that the team conducting the survey gets no answer from exactly 5 calls out of 12, if no one is at home in 15 percent of the households?

✓ We find the answer as follows: P (x=7, n=12, p =0.85) = P (x` = 5, n=12, p = 0.15) =
P (x` ≤5) - P (x` ≤ 4) = 0.9954 - 0.9761 = 0.019

P(x` =5) = P(x` ≤5) - P(x` ≤4)

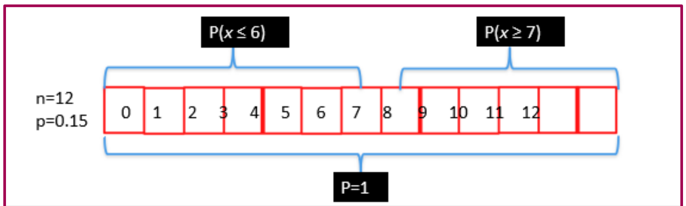
| n | x | .01 | .05 | .10 | .15 | .20 |
|---|---|---|---|---|---|---|
| 12 | 0 | 0.88638 | 0.54036 | 0.28243 | 0.14224 | 0.06872 |
| | 1 | 0.99383 | 0.88164 | 0.65900 | 0.44346 | 0.27488 |
| | 2 | 0.99979 | 0.98043 | 0.88913 | 0.73582 | 0.55835 |
| | 3 | 1.00000 | 0.99776 | 0.97436 | 0.90779 | 0.79457 |
| | 4 | 1.00000 | 0.99982 | 0.99567 | 0.97608 | 0.92744 |
| | 5 | 1.00000 | 0.99999 | 0.99946 | 0.99536 | 0.98059 |
| | 6 | 1.00000 | 1.00000 | 0.99995 | 0.99933 | 0.99610 |
| | 7 | 1.00000 | 1.00000 | 1.00000 | 0.99993 | 0.99942 |

**2.** The team will find someone at home in 5 or fewer households.
Solution: The probability we want is: The team will not find someone at home in **7 or more** households

✓ P (x≤5, n=12, p =0.85) = P (x` ≥ 7, n=12, p =0.15) =1-P (x` ≤6, n=12, p =0.15)
=1 - 0.9993 = 0.0007

P(x` ≥7)=1-P(x` ≤6 )

| n | x | .01 | .05 | .10 | .15 | .20 |
|---|---|---|---|---|---|---|
| 12 | 0 | 0.88638 | 0.54036 | 0.28243 | 0.14224 | 0.06872 |
| | 1 | 0.99383 | 0.88164 | 0.65900 | 0.44346 | 0.27488 |
| | 2 | 0.99979 | 0.98043 | 0.88913 | 0.73582 | 0.55835 |
| | 3 | 1.00000 | 0.99776 | 0.97436 | 0.90779 | 0.79457 |
| | 4 | 1.00000 | 0.99982 | 0.99567 | 0.97608 | 0.92744 |
| | 5 | 1.00000 | 0.99999 | 0.99946 | 0.99536 | 0.98059 |
| | 6 | 1.00000 | 1.00000 | 0.99995 | 0.99933 | 0.99610 |
| | 7 | 1.00000 | 1.00000 | 1.00000 | 0.99993 | 0.99942 |
| | 8 | 1.00000 | 1.00000 | 1.00000 | 0.99999 | 0.99994 |
| | 9 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| | 10 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| | 11 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| | 12 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |




| will find someone at home in 5 or fewer | will not find someone at home in 7 or more |
|---|---|
| $x \le 5$ | $x` \ge 7$ |
| $n$=12 | $n$=12 |
| $p$ =0.85 | $p$ =0.15 |

**3.** The team will find someone at home in 8 or more households.
**Solution:** The probability we desire is: The team will **not** find someone at home in **4** or **less** households. P ($x$≥8, $n$ =12, $p$ =0.85) = P ($x$`≤ 4, $n$ =12, $p$ =0.15) = 0.9761

P($x$`≤4): Direct entry!!

| n | x | .01 | .05 | .10 | .15 | .20 |
|---|---|---|---|---|---|---|
| 12 | 0 | 0.88638 | 0.54036 | 0.28243 | 0.14224 | 0.06872 |
| | 1 | 0.99383 | 0.88164 | 0.65900 | 0.44346 | 0.27488 |
| | 2 | 0.99979 | 0.98043 | 0.88913 | 0.73582 | 0.55835 |
| | 3 | 1.00000 | 0.99776 | 0.97436 | 0.90779 | 0.79457 |
| | 4 | 1.00000 | 0.99982 | 0.99567 | 0.97608 | 0.92744 |
| | 5 | 1.00000 | 0.99999 | 0.99946 | 0.99536 | 0.98059 |
| | 6 | 1.00000 | 1.00000 | 0.99995 | 0.99933 | 0.99610 |
| | 7 | 1.00000 | 1.00000 | 1.00000 | 0.99993 | 0.99942 |
| | 8 | 1.00000 | 1.00000 | 1.00000 | 0.99999 | 0.99994 |
| | 9 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |

| will find someone at home in 8 or more | will not find someone at home in 4 or less |
|---|---|
| $X \geq 8$ | $x$` ≤4 |
| $n$=12 | $n$=12 |
| $p$ =0.85 | $p$ =0.15 |

- The binomial distribution has two parameters, ***n* and *p***
- They are parameters in the sense that they are sufficient to specify a binomial distribution.
- The binomial distribution is really a family of distributions with each possible value of *n* and *p* designating a different member of the family.
- The mean and variance of the binomial distribution are:
  - ✓ $\mu = np$
  - ✓ $\sigma^2 = n*p*(1 - p)$
  - ✓ $SD = \sqrt{np(1-p)}$

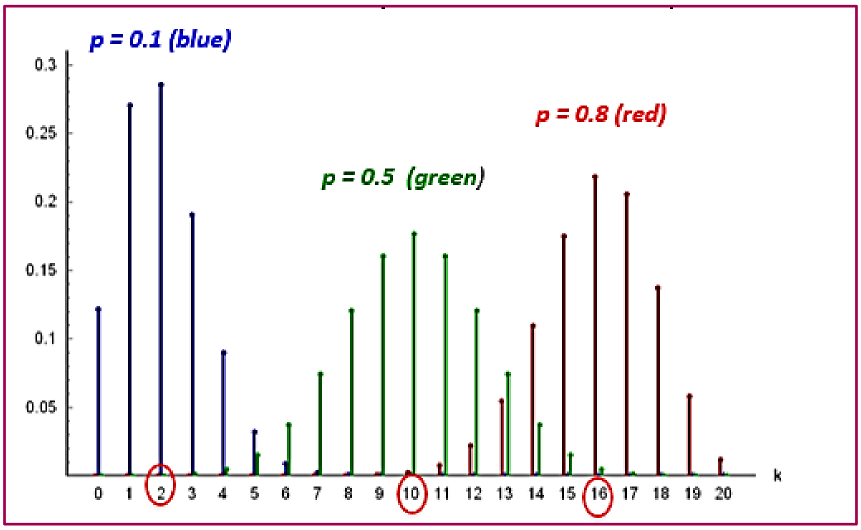
★ Example:

If n = 10 trials and the probability of success p = 0.5 then:

- ✓ $\mu = 10 * 0.5 = 5$
- ✓ $\sigma^2 = 10 * 0.5 * (1 - 0.5) = 2.5$
- ✓ $SD = \sqrt{10 * 0.5(1-0.5)} = 1.58$

- ***Binomial Distribution (mean/variance)***
  - Binomial distribution for n = 20
  - Higher n or p….> shift to the right with higher mean
  - Higher variance….> more spread (variation)

| | $\mu = np$ | $\sigma^2 = np(l - p)$ |
|---|---|---|
| 0.1 | 2 | 1.8 |
| 0.5 | 10 | 5 |
| 0.8 | 16 | 3.2 |



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